Leptogenesis in $E_6 \times U(1)_A$ SUSY GUT model

Takuya Ishihara[1][*], Nobuhiro Maekawa[1,2][†],

Mao Takegawa[1] and Masato Yamanaka[2][‡]

[1]*Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

[2]*Kobayashi Maskawa Institute, Nagoya University, Nagoya 464-8602, Japan*

Abstract

We study the thermal leptogenesis in the $E_6 \times U(1)_A$ SUSY GUT model in which realistic masses and mixings of quarks and leptons can be realized. We show that the sufficient baryon number can be produced by the leptogenesis in the model, in which the mass parameter of the lightest right-handed neutrino is predicted to be smaller than 10^8 GeV. The essential point is that the mass of the lightest right-handed neutrino can be enhanced in the model because it has a lot of mass terms whose mass parameters are predicted to be the same order of magnitude which is smaller than 10^8 GeV. We show that O(10) enhancement for the lightest right-handed neutrino mass is sufficient for the observed baryon asymmetry. Note that such mass enhancements do not change the predictions of neutrino masses and mixings at the low energy scale in the E_6 model which has six right-handed neutrinos. In the calculation, we include the effects of supersymmetry and flavor in final states of the right-handed neutrino decay. We show that the effect of supersymmetry is quite important even in the strong washout regime when the effect of flavor is included. This is because the washout effects on the asymmetries both of the muon and the electron become weaker than that of the tau asymmetry.

[*]E-mail address: ishihara@eken.phys.nagoya-u.ac.jp
[†]E-mail address: maekawa@eken.phys.nagoya-u.ac.jp
[‡]E-mail address: yamanaka@eken.phys.nagoya-u.ac.jp

1 Introduction

Supersymmetric (SUSY) grand unified theory (GUT)[1] is one of the most promising candidates as the extended model of the standard model (SM). This is because the SUSY GUT realizes two kinds of unifications, unification of the gauge interactions and unification of the matters in the SM and for both unifications, there are supports from experiments. Three gauge couplings in the SM meets at a scale, which is called the GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV. Moreover, the various hierarchies of quark and lepton masses and mixings can be naturally understood in $SU(5)$ unification if we assume that the **10** fields of $SU(5)$ induce stronger hierarchy in Yukawa couplings than the $\bar{\mathbf{5}}$ fields of $SU(5)$. One of the most important advantages of the E_6 unification[2] is that the above assumption can be naturally derived[3]. As the result of this important feature of the E_6 unification, we can build an E_6 GUT in which all three generation of quarks and leptons can be unified into a single multiplet(or two multiplets) by introducing family symmetry $SU(3)_F$(or $SU(2)_F$) and the realistic quark and lepton masses and mixings can be realized after breaking the family and GUT symmetries[4].

However, it is well-known that SUSY GUTs are suffering from the doublet-triplet splitting problem[5]. The doublet Higgs must have the weak scale mass to obtain the weak scale, while the triplet (colored) Higgs which belongs to the same multiplet as the doublet Higgs in the GUT must have the GUT scale mass to stabilize the nucleon. Fortunately, if the anomalous $U(1)_A$ gauge symmetry[6] is introduced, the problem can be solved under a natural assumption that all the interactions are introduced with $O(1)$ coefficients[7, 8, 9]. Because of this natural assumption, the coefficients of the terms and the vacuum expectation values (VEVs) of the GUT Higgs can be determined only by the symmetry of the theory. The coefficients of the interaction XYZ are determined[10, 11] except the $O(1)$coefficients by the total anomalous $U(1)_A$ charge $x + y + z$ as

$$\begin{array}{ll} \lambda^{x+y+z}XYZ & (x+y+z \geq 0) \\ 0 & (x+y+z < 0), \end{array} \tag{1}$$

where x, y, and z are the $U(1)_A$ charges of the fields X, Y, and Z, respectively. Throughout this paper, we denote all the fields with uppercase letters and their anomalous $U(1)_A$ charges with the corresponding lowercase letters if there is no special comment. Here λ is the ratio of the Fayet-Illiopoulos parameter ξ to the cutoff Λ, and in this paper we take $\lambda \sim 0.22$ as a typical value. Under the natural assumption, we can obtain the realistic Yukawa couplings in E_6 GUT[3] (or in $SO(10)$ GUT[7] which has similar structure as E_6 GUT). The VEVs of the operators O is also determined[3] by their total

anomalous $U(1)_A$ charges o as

$$\langle O \rangle = \begin{cases} 0 & (o > 0) \\ \lambda^{-o} & (o \leq 0) \end{cases}. \qquad (2)$$

In this paper, we often use a unit in which the cutoff Λ is taken to be 1. Because of the natural assumption, all the mass spectrum of superheavy particles and the VEVs of GUT Higgs are determined only by the symmetry of the theory. Therefore, we can calculate the running gauge couplings once we fix the symmetry of the theory. Interestingly, this natural scenario gives a novel explanation[9] for the experimental support for the unification of three gauge interactions in the SM. The new explanation requires that the cutoff scale must be taken to be around the usual GUT scale Λ_G[9].

If this natural E_6 GUT describes our world, it must be consistent with the cosmology. The dark matter can be the lightest supersymmetric particle. In this paper, we discuss the leptogenesis[12] in this scenario. One of the important things in E_6 unification for the leptogenesis is that the fundamental representation $\mathbf{27}$, which is decomposed in the $E_6 \supset SO(10) \times U(1)_{V'}$ notation (and in the $[SO(10) \supset SU(5) \times U(1)_V]$ notation) as

$$\mathbf{27} = \mathbf{16}_1[\mathbf{10}_1 + \bar{\mathbf{5}}_{-3} + \mathbf{1}_5] + \mathbf{10}_{-2}[\mathbf{5}_{-2} + \bar{\mathbf{5}}'_2] + \mathbf{1}'_4[\mathbf{1}'_0], \qquad (3)$$

includes two singlets $S(\mathbf{1}')$ and $N_R^c(\mathbf{1})$ under the SM gauge group, which can be the right-handed (RH) neutrinos. If we introduce three $\mathbf{27}$ for three generation quarks and leptons, we have six RH neutrinos. Basically, since the masses and Yukawa couplings of the RH neutrinos are determined by the symmetry, we can examine whether the leptogenesis works well or not in this scenario. Naively, the leptogenesis in this scenario does not work because the lightest RH neutrino becomes lighter than 10^8 GeV, i.e., this scenario looks not to satisfy the Ibarra's upper bound[13] for the lightest RH neutrino which is 10^{8-9} GeV. Actually, in a typical model, the (effective) $U(1)_A$ charges of S_i and N_{Ri}^c $(i = 1, 2, 3)$ are fixed as $(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) = (6.5, 5.5, 3.5)$ and $(\tilde{n}_{R1}^c, \tilde{n}_{R2}^c, \tilde{n}_{R3}^c) = (6, 5, 3)$, and therefore, the mass of the lightest RH neutrino S_1 becomes $M_{S_1} \sim \lambda^{13}\Lambda \sim 5.7 \times 10^7$ GeV[3]. Yukawa couplings are also easily estimated because the sum of the (effective) $U(1)_A$ charges of the up-type Higgs H_u and doublet-leptons l_i become $(\tilde{h}_u + \tilde{l}_1, \tilde{h}_u + \tilde{l}_2, \tilde{h}_u + \tilde{l}_3) = (0, -0.5, -1)$. The Yukawa couplings among l_i, S_1 and H_u become $(\lambda^{6.5}, \lambda^6, \lambda^{5.5})$. Then, we can estimate two important parameters for the leptogenesis as

$$K \equiv \Gamma_D/H \sim 40$$
$$\epsilon \equiv \frac{\Gamma(S_1 \to l + H_u) - \Gamma(S_1 \to \bar{l} + H_u^\dagger)}{\Gamma(S_1 \to l + H_u) + \Gamma(S_1 \to \bar{l} + H_u^\dagger)} \sim 5 \times 10^{-9}, \qquad (4)$$

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where Γ_D and H are the decay width of S_1 and the Hubble parameter at $T = M_{S_1}$, respectively. (In this paper we denote the lepton doublet fields with lowercase letter l in order to avoid the confusion with lepton asymmetry L in the following discussions.) Since the sufficient production of Baryon number requires $K \sim 1$ and $\epsilon \sim 10^{-7}$, this K is too large, and the ϵ is too small. The produced lepton number is calculated as

$$Y_L \equiv \frac{n_L}{s_0} \sim 10^{-13}, \tag{5}$$

which is about $O(1000)$ times smaller than the value $Y_L \sim 2.5 \times 10^{-10}$ which is required for the sufficient baryon number. Here, n_L and s_0 are the lepton number density and the entropy density today, and for simplicity, we neglect the SUSY contribution, which will be discussed later.

An important observation for leptogenesis in this scenario is that under fixed Yukawa couplings, $K \propto 1/M_{S_1}$ and $\epsilon \propto M_{S_1}$. Therefore, larger M_{S_1} results in larger baryon number. This observation is critical because in this scenario, the mass of S_1 tends to be larger than expected by the symmetry. There are two essential points in this scenario. One of them is that it has a plenty of terms which give mass to S_1. Each term gives the same order of mass to S_1 as expected by the symmetry, and the real mass can increase because of the large number of mass terms. The other point is that the predictions for the quark and lepton masses and mixings does not change so much even if the mass of S_1 becomes larger than expected by the symmetry. This is because the number of RH neutrino flavors becomes larger than three in E_6 unification. (In $SO(10)$ unification, it is not avoidable to change the predictions on neutrino sector if one of the RH neutrino masses is taken to be larger than expected by the symmetry.)

The question is how large enhancement of the mass is needed to obtain the sufficiently large baryon number. It is the main subject in this paper to answer this question.

In section 2, we briefly review the E_6 GUT with anomalous $U(1)_A$ gauge symmetry. And in section 3, we discuss the enhancement of the RH neutrino masses in this scenario. In section 4, we would like to answer the above question. First, we calculate the enhancement factor required to obtain the sufficient baryon number with simple non-SUSY Boltzmann equations. Second, we discuss the effect of lepton flavors. Third, the SUSY effect is considered. Finally, we calculate the enhancement factor, including both effects of lepton flavors and of SUSY. We show that only about O(10) enhancement of the mass of S_1 is sufficient for the observed baryon number.

	Ψ_1	Ψ_2	Ψ_3	H	$\bar{H}$	C	$\bar{C}$	A
E_6	**27**	**27**	**27**	**27**	$\overline{\bf 27}$	**27**	$\overline{\bf 27}$	**78**
$U(1)_A$	$\frac{9}{2}$	$\frac{7}{2}$	$\frac{3}{2}$	-3	1	-4	-1	-1

Table 1: Field contents of matters and GUT Higgs in a typical $E_6 \times U(1)_A$ GUT[3] and the charge assignment under $E_6 \times U(1)_A$. Here,Ψ_i ($i = 1, 2, 3$) are three generation quarks and leptons, the VEVs of H and $\bar{H}$ break E_6 into $SO(10)$, the VEVs of C and $\bar{C}$ break $SO(10)$ into $SU(5)$, and the VEV of A breaks $SU(5)$ into the standard model gauge group. The MSSM Higgs are included in H and C.

2 E_6 unification with anomalous $U(1)_A$ gauge symmetry

We briefly review the $E_6 \times U(1)_A$ GUT in this section[3]. The typical quantum numbers of fields in $E_6 \times U(1)_A$ GUT are shown in Table 1. An interesting structure in E_6 unification is that three of six $\bar{\bf 5}$ of $SU(5)$ in three matter fields $\Psi_i(\mathbf{27})$ become superheavy through the Yukawa interactions

$$(Y^H)_{ij}\Psi_i\Psi_j H + (Y^C)_{ij}\Psi_i\Psi_j C \tag{6}$$

after developing the VEVs $\langle \bar{H}H \rangle \sim \lambda^{-h-\bar{h}}$ and $\langle \bar{C}C \rangle \sim \lambda^{-c-\bar{c}}$, which break E_6 into $SO(10)$ and $SO(10)$ into $SU(5)$, respectively. Here, the components of Yukawa matrices Y^H and Y^C are fixed by the total anomalous $U(1)_A$ charges of the corresponding terms $\Psi_i\Psi_j H$ and $\Psi_i\Psi_j C$, respectively. Since the Yukawa couplings for Ψ_3 are larger than those for Ψ_2 and Ψ_1 because $\psi_3 \gg \psi_1, \psi_2$, $\bar{\bf 5}_3$ and $\bar{\bf 5}'_3$ become superheavy, and therefore, three light modes $\bar{\bf 5}$ come from the Ψ_1 and Ψ_2. This structure naturally explains why $\mathbf{10}$s of $SU(5)$ induce stronger hierarchy than $\bar{\bf 5}$s of $SU(5)$, which is important to obtain realistic hierarchies of quark and lepton masses and mixings.

The $E_6 \times U(1)_A$ GUT in Table 1 predicts the six RH neutrino masses M_α ($\alpha = 1, 2, \cdots, 6$) and the Dirac neutrino Yukawa couplings $Y_{\alpha i}$ ($i = 1, 2, 3$) as in Table 2 except $O(1)$ coefficients. In the followings, we briefly review the derivation of these predictions from the model. See Ref.[3, 9] for the detail. The masses of the RH neutrinos can be obtained through the higher dimensional interactions

$$(Y^{\bar{X}\bar{Y}})_{ij}\Psi_i\Psi_j \bar{X}\bar{Y}, \quad (\bar{X}, \bar{Y} = \bar{H}, \bar{C}), \tag{7}$$

after developing the VEVs $\langle \bar{H} \rangle \sim \lambda^{-\frac{1}{2}(h+\bar{h})}$ and $\langle \bar{C} \rangle \sim \lambda^{-\frac{1}{2}(c+\bar{c})}$. (These VEVs are determined by the VEV relations for the GUT singlet operators $\bar{H}H$ and

Table 2: GUT scale Λ_G, Majorana masses of RH neutrinos M_α ($\alpha = 1, 2, ..., 6$), and each component of neutrino Yukawa Y_ν in the $E_6 \times U(1)_A$ GUT model with $\lambda = 0.22$.

Parameter	value	comment
Λ_G	$2.000 \times 10^{16}\,\text{GeV}$	GUT scale and the cutoff scale
$M_1 = \lambda^{13}\Lambda_G$	$5.656 \times 10^7\,\text{GeV}$	1st RH neutrino mass
$M_2 = \lambda^{12}\Lambda_G$	$2.571 \times 10^8\,\text{GeV}$	2nd RH neutrino mass
$M_3 = \lambda^{11}\Lambda_G$	$1.169 \times 10^9\,\text{GeV}$	3rd RH neutrino mass
$M_4 = \lambda^{10}\Lambda_G$	$5.312 \times 10^9\,\text{GeV}$	4th RH neutrino mass
$M_5 = \lambda^7\Lambda_G$	$4.989 \times 10^{11}\,\text{GeV}$	5th RH neutrino mass
$M_6 = \lambda^6\Lambda_G$	$2.268 \times 10^{12}\,\text{GeV}$	6th RH neutrino mass
$Y_{11} = \lambda^{6.5}$	5.318×10^{-5}	11 component of Y_ν
$Y_{12} = \lambda^{6.0}$	1.134×10^{-4}	12 component of Y_ν
$Y_{13} = \lambda^{5.5}$	2.417×10^{-4}	13 component of Y_ν
$Y_{21} = \lambda^{6.0}$	1.134×10^{-4}	21 component of Y_ν
$Y_{22} = \lambda^{5.5}$	2.417×10^{-4}	22 component of Y_ν
$Y_{23} = \lambda^{5.0}$	5.154×10^{-4}	23 component of Y_ν
$Y_{31} = \lambda^{5.5}$	2.417×10^{-4}	31 component of Y_ν
$Y_{32} = \lambda^{5.0}$	5.154×10^{-4}	32 component of Y_ν
$Y_{33} = \lambda^{4.5}$	1.099×10^{-3}	33 component of Y_ν
$Y_{41} = \lambda^{5.0}$	5.154×10^{-4}	41 component of Y_ν
$Y_{42} = \lambda^{4.5}$	1.099×10^{-3}	42 component of Y_ν
$Y_{43} = \lambda^{4.0}$	2.343×10^{-3}	43 component of Y_ν
$Y_{51} = \lambda^{3.5}$	4.994×10^{-3}	51 component of Y_ν
$Y_{52} = \lambda^{3.0}$	1.065×10^{-2}	52 component of Y_ν
$Y_{53} = \lambda^{2.5}$	2.270×10^{-2}	53 component of Y_ν
$Y_{61} = \lambda^{3.0}$	1.065×10^{-2}	61 component of Y_ν
$Y_{62} = \lambda^{2.5}$	2.270×10^{-2}	62 component of Y_ν
$Y_{63} = \lambda^{2.0}$	4.840×10^{-2}	63 component of Y_ν

$\bar{C}C$ and the D-flatness conditions.) For example, the mass of S_1 (N_{R1}^c) becomes $\lambda^{2\psi_1+2\bar{h}-(h+\bar{h})}\Lambda \sim \lambda^{13}\Lambda$ $(\lambda^{2\psi_1+2\bar{c}-(c+\bar{c})}\Lambda \sim \lambda^{12}\Lambda)$. It is convenient to define the effective $U(1)_A$ charges for any fields Ψ as

$$\tilde{\psi} = \psi + \frac{1}{5}c_V(\Psi) + \frac{1}{2}c_{V'}(\Psi), \tag{8}$$

where $c_V(\Psi)$ and $c_{V'}(\Psi)$ are the $U(1)_V$ and $U(1)_{V'}$ charges of Ψ, respectively. The coefficients in the above equation (8) are determined so that the relations $\langle H \rangle \sim \lambda^{-\tilde{h}}$, $\langle \bar{H} \rangle \sim \lambda^{-\tilde{\bar{h}}}$, $\langle C \rangle \sim \lambda^{-\tilde{c}}$, and $\langle \bar{C} \rangle \sim \lambda^{-\tilde{\bar{c}}}$ are satisfied. It is obvious that the relations (1) and (2) do not change when the effective $U(1)_A$ charges are introduced because the $U(1)_V$ and $U(1)_{V'}$ charges of the E_6 invariant terms are vanishing. Although special relations between $O(1)$ coefficients due to the E_6 symmetry (or other original symmetries which are broken in the effective theory) cannot be seen explicitly in the effective model, the effective $U(1)_A$ charges are useful to estimate the couplings of any terms allowed by the original symmetry. For example, the Dirac Yukawa couplings are easily estimated by these effective $U(1)_A$ charges as $(\lambda^{\tilde{l}_i + \tilde{n}_{Rj}^c + \tilde{h}_u}, \lambda^{\tilde{l}_i + \tilde{s}_j + \tilde{h}_u})$. The mass matrices of the RH neutrinos are also calculated as $\begin{pmatrix} \lambda^{\tilde{n}_{Ri}^c + \tilde{n}_{Rj}^c} & \lambda^{\tilde{n}_{Ri}^c + \tilde{s}_j} \\ \lambda^{\tilde{s}_i + \tilde{n}_{Rj}^c} & \lambda^{\tilde{s}_i + \tilde{s}_j} \end{pmatrix}$. The RH neutrino masses M_α in Table 2 can be obtained by diagonalizing the 6×6 RH neutrino mass matrix. In Table 2 we change the ordering of the RH neutrinos' generation number α so that smaller number RH neutrino has smaller mass. The Dirac neutrino Yukawa couplings in Table 2 use this new index α. Even effective higher dimensional interactions which give the light neutrino masses can be estimated as

$$\lambda^{\tilde{l}_i + \tilde{l}_j + 2\tilde{h}_u} l_i l_j H_u^2, \tag{9}$$

which are also derived from the RH neutrino mass matrix and the Dirac neutrino Yukawa matrix by the seesaw mechanism.

One of the most interesting features in the anomalous $U(1)_A$ models is that the higher dimensional interactions give the same contributions to interactions as the lower dimensional interactions. For example, the coefficients of Yukawa interactions $\Psi_i \Psi_j H$ are determined by their total $U(1)_A$ charge as $\lambda^{\psi_i + \psi_j + h}$ except $O(1)$ coefficient. The higher dimensional interactions $\Psi_i A \Psi_j H$, whose coefficients are also determined by the total charge as $\lambda^{\psi_i + \psi_j + h + a}$, also contribute to the Yukawa interactions $\Psi_i \Psi_j H$ after developing the VEV $\langle A \rangle \sim \lambda^{-a}$ which breaks $SU(5)$ into the SM gauge group. The coefficients from the higher dimensional interactions are estimated as $\lambda^{\psi_i + \psi_j + h + a}\langle A \rangle \sim \lambda^{\psi_i + \psi_j + h}$, which is nothing but the coefficients of the original Yukawa interactions except $O(1)$ coefficients. Therefore, the unrealistic

GUT relations of Yukawa couplings, for example, $Y_d = Y_e^t$, can be naturally avoided in the anomalous $U(1)_A$ GUT models because the higher dimensional interactions with the adjoint Higgs A have different contributions to the down-type Yukawa couplings from the charged lepton Yukawa couplings after developing the VEV of A.

3 Possible enhancement for the right-handed neutrino masses

It is plausible to enhance a coefficient of an interaction if there are a lot of higher dimensional interactions which contribute to the coefficient by the same order after developing the VEVs of the negatively charged operators. Roughly, if there are N higher dimensional interactions which give the same contribution to an interaction, the enhancement factor can be expected to be $\sqrt{N}$ according to the random walk theory. Since we have introduced several negatively charged singlets as well as the GUT Higgs fields, the number N can be large if the total $U(1)_A$ charge of an interaction is large. For example, in a simplified model in which all negatively charged fields Θ_i $(i = 1, 2, \cdots, n)$ have the $U(1)_A$ charges $\theta_i = -1$, the number of the independent interactions with total $U(1)_A$ charge c is given by $N_n(c) = \frac{(n+c-1)!}{c!(n-1)!}$. This number $N_n(c)$ becomes easily large when c and n are large. For example, we obtain that $N_5(5) = 126$, $N_5(10) = 1001$, $N_{10}(10) = 92378$, $\cdots$. In this section, we will show it is plausible that the 1st, 2nd and 3rd smallest RH neutrino masses are enhanced and this enhancement does not change the physical predictions for the light neutrino sector so much.

The interactions which contribute to the masses of the RH neutrinos S_i and N_{Ri}^c $(i = 1, 2, 3)$ are $\Psi_i \Psi_i \bar{H} \bar{H}$ and $\Psi_i \Psi_i \bar{C} \bar{C}$, respectively. The total $U(1)_A$ charges of these interactions are (11, 9, 5) for S_i and (9, 7, 3) for N_{Ri}^c, while the masses expected by the symmetry are $(\lambda^{13}, \lambda^{11}, \lambda^7)$ and $(\lambda^{12}, \lambda^{10}, \lambda^6)$, respectively. This means that the enhancement factors η_{S_i} and $\eta_{N_{Ri}^c}$ for their masses are expected to be the largest for the lightest RH neutrino S_1, the second largest for the second and the third lightest neutrinos N_{R1}^c and S_2.

In this paper, we do not count the total number of the independent interactions which give the mass term of these RH neutrinos in the explicit E_6 GUT model in Table 1. However, we discuss what happens when some of the RH neutrinos have larger masses than those expected by the symmetry. It is an important observation that each RH neutrino gives the same order of the contribution to all components of the light neutrinos' mass matrix $M_\nu = Y_{\nu_D}^t M_{\nu_R}^{-1} Y_{\nu_D} \langle H_u \rangle^2$ if its mass is nothing but the value expected by

the symmetry. Therefore, if one of the enhancement factors η_{S_i} and $\eta_{N^c_{Ri}}$ is around one, all components of M_ν becomes the values expected by the symmetry, and so are all components of the diagonalizing matrix. In order to obtain three eigenvalues expected by the symmetry, three of the six enhancement factors must be around one. Then all predictions on the light neutrino sector become the same order as the predictions without the enhancement factors. Since the lightest neutrino mass has been fixed only its upper limit by experiments, the prediction for it can be different from the predicted value without any enhancement factor. Therefore, it is sufficient that two RH neutrinos have their masses which are determined by the symmetry for consistency with the present constraints obtained by neutrino experiments.

It looks not to be fair that we consider these enhancement effects only for the RH neutrino masses, although the mass terms have much larger $U(1)_A$ charges than the other terms like Yukawa terms. We should change the $U(1)_A$ charge assignment in Table 1, when such enhancement effects are taken into account. This subject is beyond the scope of this paper. Here we should emphasize that even after changing these $U(1)_A$ charges, the mass terms of S_1, S_2, and N^c_{R1} have still much larger $U(1)_A$ charges, and therefore, some enhancements for their masses are expected.

The next important question is how large enhancement factor is needed for sufficient leptogenesis in this E_6 GUT model. In the next section, we try to answer this question.

4 Leptogenesis in the $E_6 \times U(1)_A$ model

In the thermal leptogenesis scenario, thermally produced RH neutrinos go out of equilibrium as temperature decreases to their mass scale, and their CP asymmetric decays produce lepton asymmetry [12]. The lepton asymmetry is converted to the baryon asymmetry via the nonperturbative $B + L$ violating sphaleron processes [14].

In this section, we calculate the thermally produced lepton number in the $E_6 \times U(1)_A$ model with the Dirac neutrino Yukawa couplings $Y_{\alpha i}$ ($\alpha = 1, 2, \cdots, 6, i = 1, 2, 3$) which are determined by the symmetry as in Table 2 and the masses M_α for the mass eigenstate of the RH neutrinos N_α. Some of six M_α have enhancement factors η_α larger than 1. What we would like to know by this calculation is how large enhancement factors are required to obtain sufficiently large lepton number. In the calculation, it is important to include supersymmetric contributions and the effects of lepton flavor in the final state of the decay process simultaneously. To show this statement, we calculate the sufficient enhancement factor in four cases:
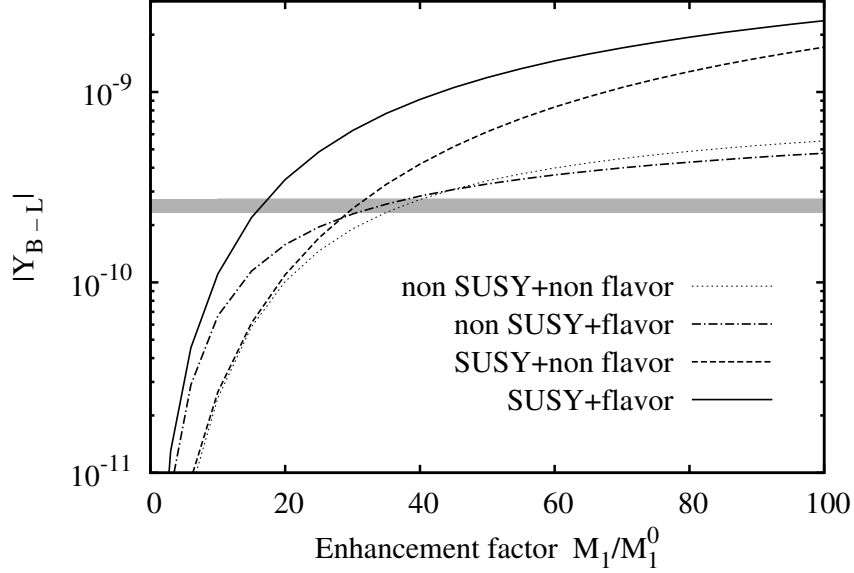
Figure 1: $\eta_1 \equiv M_1/M_1^0$ dependence of $|Y_{B-L}|$ in each case. Horizontal band corresponds to the observed baryon asymmetry in SUSY cases. M_1^0 is the "bare" Majorana mass in the absence of the $U(1)_A$ interactions. We take the simplified CP asymmetry $\epsilon_1^{\rm SM}(13)$, $\epsilon_{1i}^{\rm SM}$ (16), $\epsilon_1^{\rm SUSY} = 2 \times \epsilon_1^{\rm SM}$, and $\epsilon_{1i}^{\rm SUSY} = 2 \times \epsilon_{1i}^{\rm SM}$ with the assumption $\Im[(Y^\dagger Y)_{61}] = \Re[(Y^\dagger Y)_{61}]$ for the calculation in each case, respectively. $K^{\rm SM}$ and $K^{\rm SUSY}$ can be written as $K^{\rm SM} \sim 37/\eta_1$ and $K^{\rm SUSY} \sim 51/\eta_1$, respectively.

- non-SUSY + non flavor
- non-SUSY + flavor
- SUSY + non flavor
- SUSY + flavor

The result is shown in Fig. 1. In a realistic situation of the $E_6 \times U(1)_A$ GUT model, i.e., in the case of SUSY+flavor, the sufficient lepton number can be obtained if the enhancement factor for the N_1 mass is around 16. This means that $M_1 \sim 9 \times 10^8$ GeV.

It is known that supersymmetric contribution is important when K is smaller than 1 because supersymmetric calculation makes K larger effectively. On the other hand, the lepton flavor effects are important when the decay parameter $K \equiv \Gamma_{N_1}(T = 0)/H(T = M_1)$ is larger than 1 because K
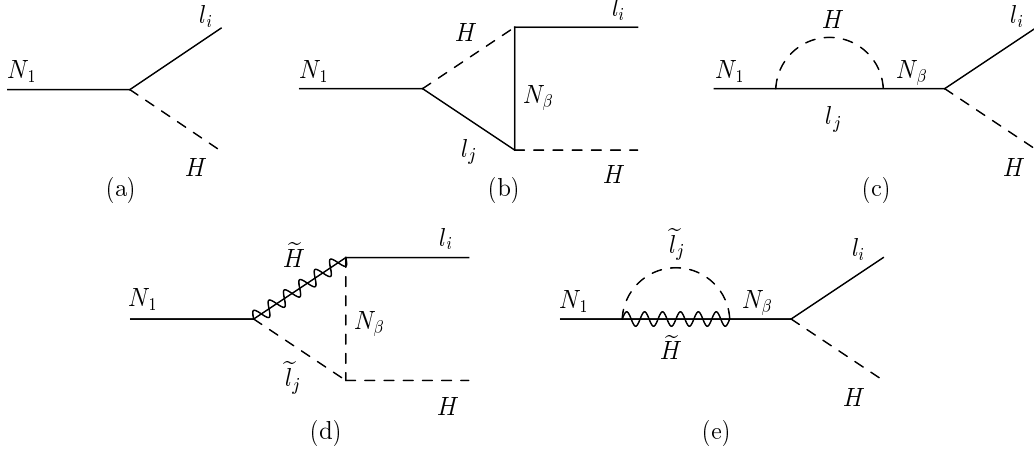
10

Figure 2: Tree and one-loop diagrams contributing to the CP asymmetric decay of lightst RH neutrino. In the model, five RH neutrinos N_β ($\beta = 2, ..., 6$) contribute to the CP asymmetry.

for the muon and the electron become smaller than K for the tau. Here T is temperature of the universe. Our calculation shows that it is important to inculude both contributions when K is larger than 1. This is because supersymmetric contribution is important for smaller K of the electron and the muon.

4.1 non-SUSY + non-flavor

In this subsection, we evaluate the lepton asymmetry in the non-SUSY+non-flavor case in $E_6 \times U(1)_A$ models after brief review. This simple calculation is important to understand the outline of the thermal leptogenesis with the RH neutrino masses and the Dirac Yukawa couplings in Table 2.

In the model, since RH neutrinos are hierarchical in mass, the lepton asymmetry is generated by the CP asymmetric reactions of the lightest RH neutrino N_1. In the followings, we assume that the lightest RH neutrino is N_1 while it has the largest enhancement factor. The lepton asymmetry is evaluated by a coupled set of evolution equations of the lightest RH neutrino N_1 and the lepton asymmetry L:

$$\frac{dY_{N_1}}{dz} = -\frac{z}{sH(z=1)} \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) [\gamma_D + 2\gamma_{Ss} + 4\gamma_{St}], \qquad (10)$$
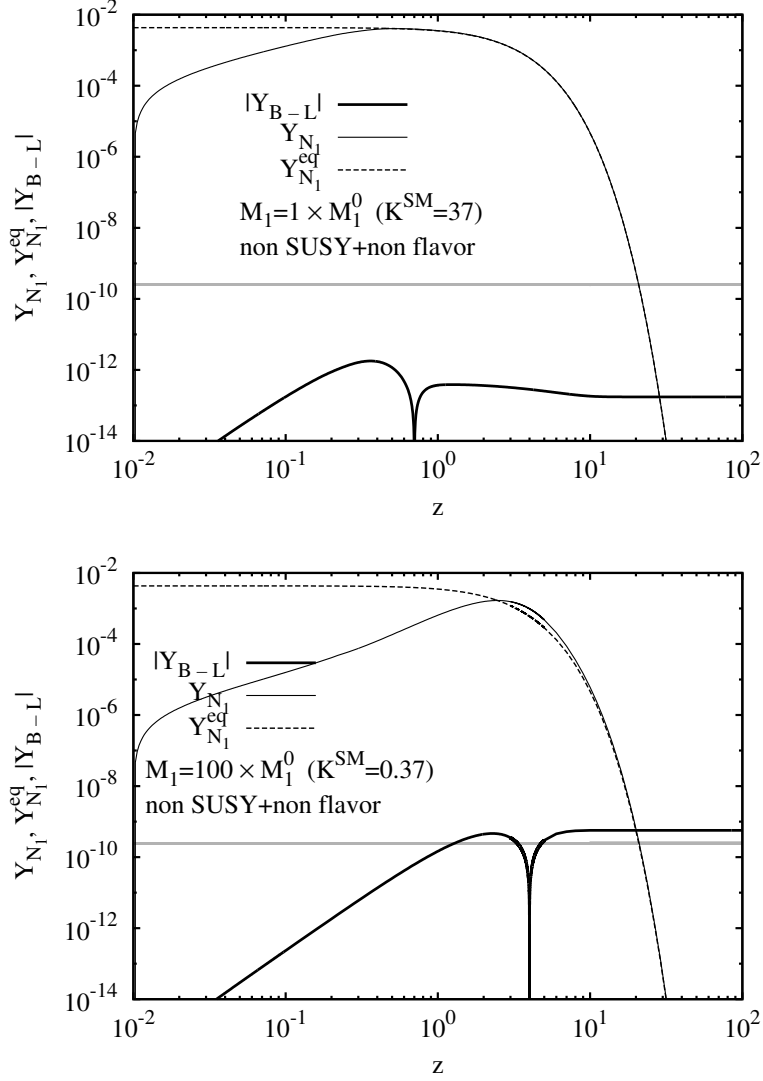
Figure 3: Evolutions of $|Y_{B-L}|$, Y_{N_1}, and $Y_{N_1}^{eq}$ for $M_1 = 1 \times M_1^0$ (top panel) and $M_1 = 100 \times M_1^0$ (bottom panel) in the non-SUSY+non-flavor case. Here M_1^0 is the "bare" Majorana mass in the absence of the $U(1)_A$ interactions. Horizontal band corresponds to the observed baryon asymmetry. We take the simplified CP asymmetry (13) with the assumption $\Im[(Y^\dagger Y)_{61}] = \Re[(Y^\dagger Y)_{61}]$.

$$\frac{dY_L}{dz} = -\frac{z}{sH(z=1)} \left\{ \left[\frac{1}{2}\frac{Y_L}{Y_l^{eq}} + \epsilon_{N_1}^{SM} \left(1 - \frac{Y_{N_1}}{Y_{N_1}^{eq}} \right) \right] \gamma_D + \frac{Y_L}{Y_l^{eq}} \left[2\gamma_{St} + \frac{Y_{N_1}}{Y_{N_1}^{eq}}\gamma_{Ss} \right] \right\}.$$

(11)

Here s is the entropy density, and H is the Hubble parameter. We use

a dimensionless variable $z \equiv M_1/T$. Y_X and Y_X^{eq} are yield value and its equilibrium one of a species X, respectively, which are the number density normalized to the entropy density. $\gamma_D = \gamma_D(N_1 \leftrightarrow lH)$, $\gamma_{Ss} = \gamma_{Ss}(N_1 l \leftrightarrow Q_3 T_R^c)$, and $\gamma_{St} = \gamma_{St}(N_1 Q_3 \leftrightarrow lT_R)$ are thermal averaged decay rate (inverse decay rate), s-channel, and t-channel scattering rate, respectively [22]. Here l, Q_3, and T_R denote $SU(2)_L$ doublet lepton, third generation doublet quark, and singlet top quark, respectively. The CP asymmetry $\epsilon_{N_1}^{\rm SM}$ is defined as $\epsilon_{N_1}^{\rm SM} = [\Gamma(N_1 \to lH) - \Gamma(N_1 \to \bar{l}H^\dagger)]/[\Gamma(N_1 \to lH) + \Gamma(N_1 \to \bar{l}H^\dagger)]$. The first non-zero contribution to $\epsilon_{N_1}^{\rm SM}$ comes from interference between tree-level amplitude with the one-loop contributions (upper three diagrams in Fig. 2), and it is calculated in a hierarchical limit in RH neutrino masses as $\epsilon_{N_1}^{\rm SM} = -(3/16\pi) \sum_{\beta \neq 1}^6 \left(\Im\left[(Y^\dagger Y)_{\beta 1}^2 \right] / \left[Y^\dagger Y \right]_{11} \right) (M_1/M_\beta)$ [15]. Note that, the E_6 GUT model has six RH neutrinos, and therefore, $\beta = 2, 3, \cdots, 6$.

Key ingredients for the lepton asymmetry generation are the CP asymmetry $\epsilon_{N_1}^{\rm SM}$ and the decay parameter $K \equiv \Gamma_{N_1}(T = 0)/H(T = M_1)$ [16] which parametrizes the departure from the thermal equilibrium of RH neutrinos at $T = M_1$. K is important because it is related with γ_D and the factor $(1 - Y_{N_1}/Y_{N_1}^{eq})$ in Eq. (11). The lepton asymmetry Y_L is essentially determined by the above two parameters as

$$Y_L \sim \epsilon_{N_1}^{\rm SM} C(K). \tag{12}$$

The behavior of the function $C(K)$ is as follows. When $K > 1$, $C(K)$ becomes a decreasing function of K. $K > 1$ means that the RH neutrinos are still in the thermal equilibrium at $T = M_1$, and therefore, the number density of N_1 decreases rapidly when $T < M_1$. This reduces the produced lepton asymmetry. Obviously larger K results in lower decoupling temperature, smaller Y_{N1} after the decoupling and smaller lepton asymmetry. When $K < 1$, $C(K)$ becomes a increasing function of K. $K < 1$ means that the RH neutrinos are out of the thermal equilibrium at $T = M_1$, and therefore, the number of thermally produced RH neutrinos becomes smaller for smaller K. This reduces the produced lepton asymmetry. Around $K \sim 1$, the function $C(K)$ becomes maximal. Sufficient lepton asymmetry can be obtained when $K \sim 1$ and $\epsilon_{N_1}^{\rm SM} \sim 10^{-7}$.

Let us calculate the above two important parameters in the $E_6 \times U(1)_A$ GUT. First, we estimate $\epsilon_{N_1}^{\rm SM}$ and $K^{\rm SM}$ without any enhancement factor η_α

as

$$\epsilon_{N_1}^{\rm SM} = 2\left(-\frac{3}{16\pi}\frac{\Im\left[\left(Y^\dagger Y\right)_{61}^2\right]}{[Y^\dagger Y]_{11}}\frac{M_1}{M_6}\right) \sim -8.85\times10^{-9}\left(\frac{M_1}{5.7\times10^7\,{\rm GeV}}\right) \quad (13)$$

$$K^{\rm SM} = \frac{[Y^\dagger Y]_{11}M_1/8\pi}{1.66(g_*^{\rm SM})^{1/2}M_1^2/M_{\rm pl}} \simeq 37\left(\frac{5.7\times10^7\,{\rm GeV}}{M_1}\right), \quad (14)$$

where $M_{\rm pl}$ is the Planck scale and $g_*^{\rm SM}$ is the effective relativistic degrees of freedom, which is obtained as $g_*^{\rm SM} = 106.75$ with the SM particle contents. For the estimation of $\epsilon_{N_1}^{\rm SM}$, we have adopted two assumptions. The first assumption is that $\Im\left[\left(Y^\dagger Y\right)_{\beta 1}^2\right]$ can be estimated by $\Re\left[\left(Y^\dagger Y\right)_{\beta 1}^2\right]$. This assumption is reasonable because we regard all Yukawa couplings as complex numbers. The second assumption is on the factor 2 in front of the parenthesis in Eq. (13). An important observation is that $\left(Y^\dagger Y\right)_{\beta 1}^2\left[Y^\dagger Y\right]_{11}^{-1}\frac{M_1}{M_\beta} \sim \lambda^{11}$ is not dependent on β. Therefore, we can expect an enhancement factor after summation of the index β, and we assume that the enhancement factor is two through all calculations in this paper.

It is obvious that the lepton asymmetry with these parameters are too small to explain the observation. $K^{\rm SM}$ is too large and $\epsilon_{S_1}^{\rm SM}$ is too small. However, as discussed in the previous section, the lightest RH neutrino mass can be expected to have an enhancement factor which can be much larger than one. Interestingly, when the lightest RH neutrino mass M_1 becomes larger, the produced lepton asymmetry becomes larger because the CP asymmetry $\epsilon_{N_1}^{\rm SM}$ becomes larger and the decay parameter $K^{\rm SM}$ becomes smaller as seen in Eq. (13). For example, if we take the enhancement factor is around 37, the sufficient lepton asymmetry can be expected because $K^{\rm SM} \sim 1$ and $\epsilon_{N_1}^{\rm SM} \sim 3\times10^{-7}$.

Figure 3 shows the evolutions of the lepton asymmetry $|Y_{B-L}|$, yield value of the RH neutrino Y_{N_1}, and its equilibrium one $Y_{N_1}^{eq}$ for $M_1 = 1\times M_1^0$ (top panel) and $M_1 = 100\times M_1^0$ (bottom panel). Here M_1^0 represents the "bare" Majorana mass, that is the physical mass of the lightest RH neutrino without any enhancement factor. The lepton asymmetry for $M_1 = 1\times M_1^0$, $|Y_L| \simeq 10^{-13}$, is too small to account for the observed baryon asymmetry. In non-SUSY cases, the required lepton asymmetry is $2.285\times10^{-10} \le Y_{B-L} \le 2.685\times10^{-10}$ with the conversion rate of the lepton asymmetry to the baryon asymmetry $Y_B = (28/79)Y_{B-L}$ [18] and the observed baryon number $8.097\times10^{-11} \le Y_B \le 9.518\times10^{-11}$ [17]. For $M_1 = 100\times M_1^0$, the lepton asymmetry is drastically enhanced. The enhancement of the lightest RH neutrino mass makes the CP asymmetry larger [see Eq. (13)] and reduces the $K^{\rm SM}$ factor. Indeed, in the bottom panel in Fig. 3, we find the larger deviation from

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thermal equilibrium compared with the top panel. The combination of these effects leads the enhancement of lepton asymmetry.

Dotted line in Fig. 1 shows the M_1 dependence of the lepton asymmetry in the non-SUSY+non-flavor case. Since numerically $C(K) \propto K$ for $K \ll 1$, the lepton asymmetry becomes asymptotically a constant for the enhancement factor larger than 40. In this case, the physical mass of the RH neutrino is required to be $M_1 = (35 - 39) \times M_1^0$ to account for the observed baryon number.

4.2　non-SUSY + flavor

It is important for the evaluation of lepton asymmetry in the $E_6 \times U(1)_A$ model to separately involve each lepton flavor channel of the CP asymmetric decays. The reasons are as follows. The E_6 model possesses the features: (i) the evolution of lightest RH neutrino is in the strong washout regime, (ii) all of asymmetry productions of each lepton flavor by the CP asymmetric decay are sizable. These features can give rise to $\mathcal{O}(1)$ corrections to the final lepton asymmetry with respect to the case where the flavor effects are ignored [20]. This is because that the evolutions of the lepton asymmetries of each lepton flavor are in the regime of washout with different magnitudes. In this section we briefly review the leptogenesis with the flavor effects.

The total lepton asymmetry is given by the sum of the asymmetry of each lepton flavor, $Y_{B-L} = Y_{\Delta_e} + Y_{\Delta_\mu} + Y_{\Delta_\tau}$, where $\Delta_i = B/3 - L_i$. The subscript i represents the lepton flavor. The evolutions of each asymmetry is described by the flavor dependent Boltzmann equations,

$$\frac{dY_{\Delta_i}}{dz} = -\frac{z}{sH(z=1)}\left\{ \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\epsilon_{1i}^{\mathrm{SM}}\gamma_D + K_i^0 \sum_j \left[\frac{1}{2}\left(C_{ij}^l + C_j^H\right)\gamma_D \right.\right.$$
$$\left.\left. + \left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\left(C_{ij}^l \gamma_{S_s} + \frac{C_j^H}{2}\gamma_{S_t}\right) + \left(2C_{ij}^l + C_j^H\right)\left(\gamma_{S_t} + \frac{\gamma_{S_s}}{2}\right)\right]\frac{Y_{\Delta_i}}{Y_l^{eq}}\right\}. \tag{15}$$

The coefficient K_i^0 is the flavor projection, $K_i^0 = Y_{1i}Y_{1i}^*/(YY^\dagger)_{11}$. The flavor dependent CP asymmetry is defined as $\epsilon_{1i}^{\mathrm{SM}} = [\Gamma(N_1 \to l_i H) - \Gamma(N_1 \to \bar{l}_i H^\dagger)]/[\Gamma(N_1 \to l_i H) + \Gamma(N_1 \to \bar{l}_i H^\dagger)]$, and is calculated in the hierarchical mass limit as $\epsilon_{1i}^{\mathrm{SM}} = -\left(1/8\pi(YY^\dagger)_{11}\right)\sum_{\beta\neq1}^{6}\Im\left\{Y_{\beta i}Y_{1i}^*\left[(3/2)(M_1/M_\beta)(YY^\dagger)_{\beta1} + \left(M_1^2/M_\beta^2\right)(YY^\dagger)_{1\beta}\right]\right\}$ [15]. We follow the considerations for deriving Eq. (13), and obtain the simplified CP asymmetry in the non-SUSY+flavor case as

$$\epsilon_{1i}^{\mathrm{SM}} = 2\left(\frac{-1}{8\pi(YY^\dagger)_{11}}\Im\left\{Y_{6i}Y_{1i}^*\left[\frac{3}{2}\frac{M_1}{M_6}(YY^\dagger)_{61} + \frac{M_1^2}{M_6^2}(YY^\dagger)_{16}\right]\right\}\right). \tag{16}$$

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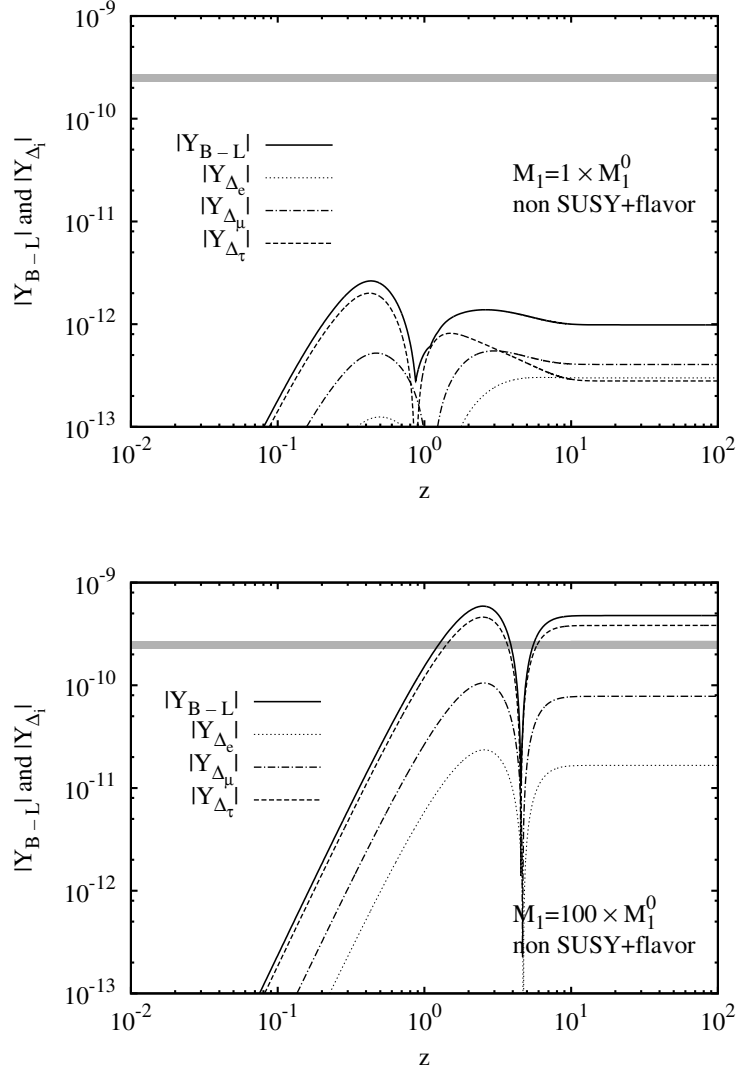Figure 4: Same as Fig. 3, but in the non-SUSY+flavor case. The evolutions of electron number L_e, muon number L_μ, and tau number L_τ are also plotted. Since the evolution of RH neutrino is same as the non-SUSY+non-flavor case, we cut the part.

The coefficient C^l (C^H) in Eq. (15) is introduced as the conversion factor between the asymmetry normalized to the equilibrium number density for l_i (H) and the yield value of each lepton number normalized to equilibrium lepton density as $\left(n_{l_i} - n_{\bar{l}_i}\right)/n_{l_i}^{\mathrm{eq}} = -\sum_j C_{ij}^l \left(Y_{\Delta_j}/Y_l^{\mathrm{eq}}\right)$, and $(n_H - n_{\bar{H}})/n_H^{\mathrm{eq}} = -\sum_j C_j^H \left(Y_{\Delta_j}/Y_l^{\mathrm{eq}}\right)$. The entries are model-independent, which are deter-

mined by constraints among the chemical potentials enforced by the equilibrium reactions in the temperature $T \sim M_1$ where the asymmetries are generated. The region of RH neutrino mass we consider is $1 \leq M_1/M_1^0 \leq 100$, and the relevant temperature of the leptogenesis in the model is in $10^5 \, \mathrm{GeV} \lesssim T \lesssim 10^{11} \, \mathrm{GeV}$. In this range, SM gauge interactions, third and second generation Yukawa interactions are in equilibrium, and the equilibrium conditions lead to the following C^l and C^H [20],

$$C_{ij}^l = \frac{1}{2148} \begin{pmatrix} 906 & -120 & -120 \\ -75 & 688 & -28 \\ -75 & -28 & 688 \end{pmatrix}, \quad C^H = \frac{1}{358} \begin{pmatrix} 37 & 52 & 52 \end{pmatrix}. \quad (17)$$

Figure 4 shows the evolutions of total lepton asymmetry $|Y_{B-L}|$, and of the asymmetry of each lepton flavor $|Y_{\Delta_i}|$ $(i = e, \mu, \tau)$ for $M_1 = 1 \times M_1^0$ (top panel) and for $M_1 = 100 \times M_1^0$ (bottom panel). For the interpretation of the result, we need to see both the magnitude of the washout and the production efficiency of each lepton asymmetry. We rearrange the K^{SM} factor (14) to involve the flavor dependence as $K_i^{\mathrm{SM}} = K_i^0 K^{\mathrm{SM}}$. Each K_i^{SM} is obtained as follows,

$$K_e^{\mathrm{SM}} = \frac{\Gamma_{N_1 \to l_e H}^{\mathrm{SM}}(T=0)}{H(T=M_1)} \simeq 1.4 \left(\frac{5.7 \times 10^7 \, \mathrm{GeV}}{M_1} \right), \quad (18)$$

$$K_\mu^{\mathrm{SM}} = \frac{\Gamma_{N_1 \to l_\mu H}^{\mathrm{SM}}(T=0)}{H(T=M_1)} \simeq 6.4 \left(\frac{5.7 \times 10^7 \, \mathrm{GeV}}{M_1} \right), \quad (19)$$

$$K_\tau^{\mathrm{SM}} = \frac{\Gamma_{N_1 \to l_\tau H}^{\mathrm{SM}}(T=0)}{H(T=M_1)} \simeq 29 \left(\frac{5.7 \times 10^7 \, \mathrm{GeV}}{M_1} \right). \quad (20)$$

The K_i^{SM} is a measure of magnitude of the washout of each lepton asymmetry, that is the same with the relation between the K^{SM} factor and the washout of total lepton asymmetry. On the other hand, the ratio of asymmetry productions of each lepton flavor by the CP asymmetric decay is equal to the ratio of each CP asymmetry, and is obtained as follows

$$\begin{aligned} \frac{L_e \text{ production}}{L_\tau \text{ production}} &= \frac{\Im [Y_{61} Y_{11}^*]}{\Im [Y_{63} Y_{13}^*]} = 0.048, \\ \frac{L_\mu \text{ production}}{L_\tau \text{ production}} &= \frac{\Im [Y_{62} Y_{12}^*]}{\Im [Y_{63} Y_{13}^*]} = 0.220. \end{aligned} \quad (21)$$

Here we assumed $\Im [Y_{6i} Y_{1i}^*] = \Re [Y_{6i} Y_{1i}^*]$. For $M_1 = 1 \times M_1^0$, K_e and K_μ are $\mathcal{O}(1)$, but $K_\tau \simeq 29$. Each lepton asymmetry is generated by the CP asymmetric decays, and then a large part of the tau's is washed out by the inverse

decays and so on, while the electron's and the muon's soon decouple from the equilibrium and survive. Thus, nonetheless the production efficiencies of the electron and the muon number are lower than the tau's, they yields a large part of the lepton asymmetry. On the other hand, for $M_1 = 100 \times M_1^0$, $K_\tau \simeq \mathcal{O}(10^{-1})$, and K_e, $K_\mu \simeq \mathcal{O}(10^{-2})$. These K_i factors indicate that each lepton number generated by the CP asymmetric decay survives without being strongly washed out. Thus a large part of lepton asymmetry is governed by the tau's.

The chain line in Fig. 1 shows the M_1 dependence of the total lepton asymmetry in the non-SUSY+flavor case. In the non-SUSY+flavor case, the physical mass of the RH neutrino is required to be $30 \leq M_1/M_1^0 \leq 37$ to account for the observed baryon number. For $M_1/M_1^0 \lesssim 40$, in the non-SUSY+non-flavor case, the evolution of the lepton asymmetry is in the strong washout regime. While, in the non-SUSY+flavor case, the muon and the electron asymmetries are not hardly washed out, and yield sizable contribution to total lepton asymmetry. The lepton asymmetry with the flavor effects is therefore larger than that in the case where the effects are ignored. On the other hand, for $M_1/M_1^0 \gtrsim 40$, the evolutions of the asymmetries of all lepton flavor are in the weak washout regime, and hence total lepton asymmetry is determined by only the asymmetry production by the CP asymmetric decay. Due to the additional washout contributions, in the parameter region, the final lepton asymmetry can be smaller than that the case without the flavor effects [19, 20].

4.3   SUSY + non-flavor

The SUSY extension of the leptogenesis gives an enhancement for the lepton asymmetry which is roughly estimated as

$$\frac{Y_B^{\text{SUSY}}}{Y_B^{\text{SM}}} \sim \begin{cases} \sqrt{2} & \text{(strong washout)} \\ 2\sqrt{2} & \text{(weak washout)} \end{cases} \tag{22}$$

in Ref. [21]. In this section, we briefly review the corrections to interpret the numerical results in the context of the $E_6 \times U(1)_A$ model.

We have two important points which increase the lepton asymmetry. The additional decay channels correct the definition of the CP asymmetry as $\epsilon_{N_1}^{\text{SUSY}} = \left[\Gamma(N_1 \to lH) - \Gamma(N_1 \to \bar{l}H^\dagger) + \Gamma(N_1 \to \tilde{l}\tilde{H}) - \Gamma(N_1 \to \tilde{l}^* \bar{\tilde{H}}) \right]/\Gamma_{N_1}^{\text{SUSY}}$. Similarly, for the RH sneutrino, $\epsilon_{\tilde{N}_1}^{\text{SUSY}} = \left[\Gamma(\tilde{N}_1 \to \tilde{l}H) - \Gamma(\tilde{N}_1 \to \tilde{l}^* H^\dagger) + \Gamma(\tilde{N}_1 \to l\tilde{H}) - \Gamma(\tilde{N}_1 \to \bar{l}\bar{\tilde{H}}) \right]/\Gamma_{\tilde{N}_1}^{\text{SUSY}}$. Here $\Gamma_{\tilde{N}_1}^{\text{SUSY}}$ is the total width of the RH sneutrino. The CP asymmetry receives the contributions of not only the RH neutrinos also of its scalar partner, and is obtained as $\epsilon_{N_1}^{\text{SUSY}} =$

$-(3/8\pi)\sum_{\beta\neq 1}^{6}\left(\Im\left[\left(Y^{\dagger}Y\right)^{2}_{\beta 1}\right]/\left(Y^{\dagger}Y\right)_{11}\right)(M_1/M_\beta)$ in the hierarchical limit of RH neutrino masses [15]. The CP asymmetry of the RH sneutrino is equal to that of the RH neutrino in the hierarchical mass limit. Repeating the consideration for deriving Eq. (13), we obtain the simplified CP asymmetries in the SUSY+non-flavor case as follows,

$$\epsilon^{\text{SUSY}}_{N_1} = \epsilon^{\text{SUSY}}_{\tilde{N}_1} = 2\left(-\frac{3}{8\pi}\frac{\Im\left[\left(Y^{\dagger}Y\right)^{2}_{61}\right]}{\left(Y^{\dagger}Y\right)_{11}}\frac{M_1}{M_6}\right) = 2\times\epsilon^{\text{SM}}_{N_1}. \tag{23}$$

These effects make the lepton asymmetry four times larger.

However, the effective relativistic degrees of freedom $g^{\text{SUSY}}_* = 228.75$ is about twice of g^{SM}_*, which reduces the lepton asymmetry to entropy ratio by $1/2$.

The most important one is the correction of K factor. In the context of SUSY, there exists additional decay channels of the RH neutrino, and the total width is obtained as $\Gamma^{\text{SUSY}}_{N_1} = \Gamma(N_1 \to lH) + \Gamma(N_1 \to \bar{l}H^{\dagger}) + \Gamma(N_1 \to \tilde{l}\tilde{H}) + \Gamma(N_1 \to \tilde{l}^*\bar{\tilde{H}}) = [Y^{\dagger}Y]_{11}M_1/4\pi = 2\times\Gamma^{\text{SM}}_{N_1}$. $\tilde{l}$ and $\tilde{H}$ represent SUSY partners of the $SU(2)_L$ lepton and the Higgs doublet, respectively. The K factor is calculated as follows,

$$K^{\text{SUSY}} = \frac{\Gamma^{\text{SUSY}}_{N_1}(T=0)}{H(T=M_1)} = \frac{[Y^{\dagger}Y]_{11}M_1/4\pi}{1.66(g^{\text{SUSY}}_*)^{1/2}M_1^2/M_{\text{pl}}} \simeq 51\left(\frac{5.7\times 10^7\,\text{GeV}}{M_1}\right). \tag{24}$$

Roughly, the factor K^{SUSY} is $\sqrt{2}$ times larger than that in the SM. This effect reduces the lepton asymmetry in strong washout regime and enhances it in weak washout regime.

Finally we note the conversion rate from the lepton asymmetry to the baryon asymmetry. In the context of SUSY, the additional equilibrium reactions at the temperature $T \simeq M_1$ alter the constraints among the chemical potentials. The alteration leads the conversion rate as $Y_B = (8/23)Y_{B-L}$ [24]. Consequently the required lepton asymmetry in SUSY cases is $2.328\times 10^{-10} \lesssim |Y_{B-L}| \lesssim 2.736\times 10^{-10}$.

With all these effects, the result in Eq. (22) is obtained.

Dashed line in Fig. 1 shows the M_1 dependence of the lepton asymmetry in the SUSY+non-flavor case. The lepton asymmetry is given by a sum of partial asymmetries from the CP asymmetric decays of RH neutrino and its scalar partner. It is evaluated by a coupled set of evolution equations of the RH neutrino, its scalar partner, and the partial asymmetries [22]. Due to too strong washout, for $M_1 \lesssim 30\times M_1^0$, nonetheless the additional CP asymmetric decays, the lepton asymmetry is close to that in the non-SUSY+non-flavor
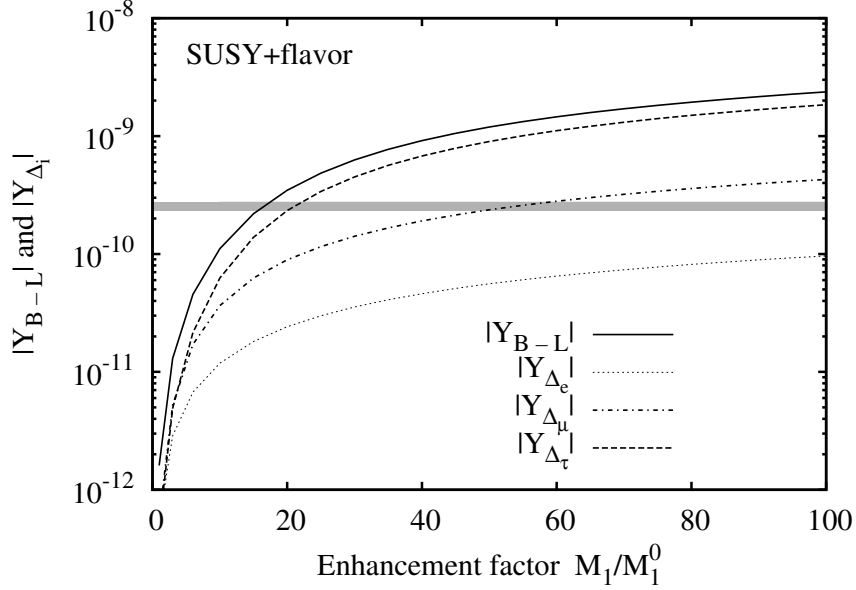
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Figure 5: $\eta_1 \equiv M_1/M_1^0$ dependence of $|Y_{B-L}|$ and $|Y_{\Delta_i}|$ in the SUSY+flavor case. Horizontal band corresponds to the observed baryon asymmetry. We take the simplified CP asymmetry $\epsilon_{1i}^{\rm SUSY} = 2 \times \epsilon_{1i}^{\rm SM}$ with the assumption $\Im[(Y^\dagger Y)_{61}] = \Re[(Y^\dagger Y)_{61}]$, where $\epsilon_{1i}^{\rm SM}$ is given by (16). $K^{\rm SUSY}$ can be written as $K^{\rm SUSY} \sim 51/\eta_1$.

case. While, for $M_1 \gtrsim 30 \times M_1^0$, because of both $K \simeq \mathcal{O}(1)$ and the additional CP asymmetric decays, larger lepton asymmetry is generated than those in the cases of non-SUSY+non-flavor and non-SUSY+flavor.

4.4   SUSY + flavor

We are now in a position to discuss the lepton asymmetry in the SUSY+flavor case, which involves the SUSY particles contributions with the flavor effects [see Sec. 4.2]. This is the realistic situation in the $E_6 \times U(1)_A$ model. Interestingly, even in the strong washout regime which is defined by $K^{\rm SM} > 1$, the effect of SUSY becomes sizable if the flavor effects are included.

The sum of total lepton and slepton asymmetries, $Y_L^{(f)}$ and $Y_L^{(s)}$, converts to the baryon asymmetry, and which are given by the sum of the asymmetry of each lepton and slepton flavor: $Y_{B-L} = Y_L^{(f)} + Y_L^{(s)} = (Y_{\Delta_e} + Y_{\Delta_\mu} + Y_{\Delta_\tau}) + (Y_{\tilde{\Delta}_e} + Y_{\tilde{\Delta}_\mu} + Y_{\tilde{\Delta}_\tau})$. We take the SUSY spectrum to be $\mathcal{O}(1\,{\rm TeV})$ in the $E_6 \times U(1)_A$ model. Then, throughout the temperature region we consider, the
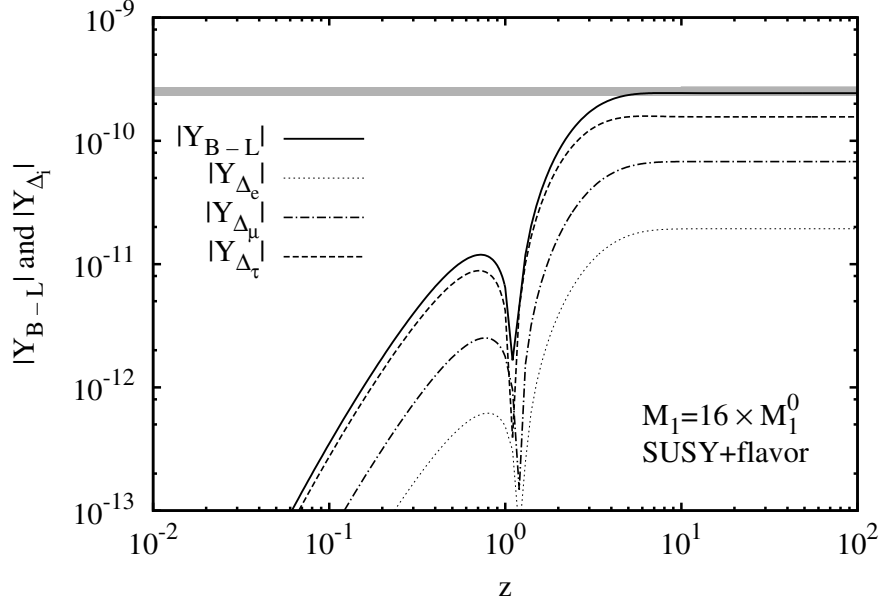
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Figure 6: Evolutions of $|Y_{B-L}|$ and $|Y_{\Delta_i}|$ for $M_1 = 16 \times M_1^0$ in the SUSY+flavor case. Horizontal band corresponds to the observed baryon asymmetry. We take the simplified CP asymmetry $\epsilon_{1i}^{\mathrm{SUSY}} = 2 \times \epsilon_{1i}^{\mathrm{SM}}$ with the assumption $\Im[(Y^\dagger Y)_{61}] = \Re[(Y^\dagger Y)_{61}]$, where $\epsilon_{1i}^{\mathrm{SM}}$ is given by (16).

equality of chemical potentials of a SM particle and its superpartner, which is refferd as superequilibration [25], is maintained in the presence of equilibrium supergauge (or Yukawa) reactions. In the superequilibrium regime, since the equality of asymmetries of each lepton and its scalar partner is also maintained, $Y_{B-L} = 2 \times (Y_{\Delta_e} + Y_{\Delta_\mu} + Y_{\Delta_\tau})$. Thus the baryon asymmetry in the case is evaluated by a coupled set of evolution equations of the RH neutrino, its scalar partner, and asymmetry of each lepton flavor. The flavor dependent Boltzmann equations are shown in Ref. [23], and relevant cross sections are given in Ref. [22].

Figure 5 shows the M_1 dependence of the lepton asymmetry $|Y_{B-L}|$ and the partial asymmetry of each lepton flavor $|Y_{\Delta_i}|$. From the Fig. 5, the enhancement factor $16 \le M_1/M_1^0 \le 17$ can yield the observed baryon number. The M_1 dependence of the asymmetries are described by the washout effects and production efficiencies of the asymmetries, that is basically the same as in the non-SUSY+flavor case. The magnitude of washout of each lepton flavor is parametrized by K_i^{SUSY}, which is SUSY extension of K_i^{SM} [Eqs. (18)
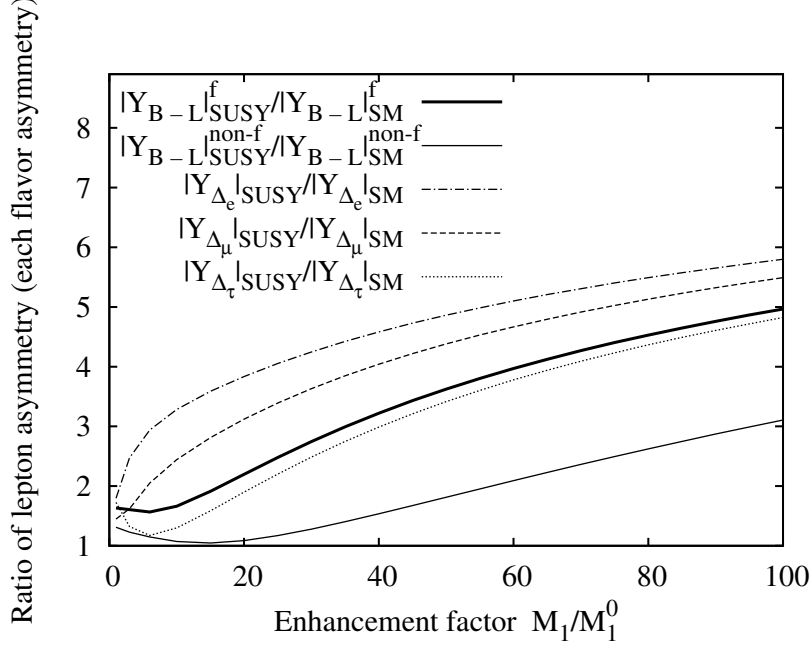
Figure 7: $\eta_1 \equiv M_1/M_1^0$ dependence of the ratio of flavor contributions in each case, $(|Y_{\Delta_i}|)_{\text{SUSY}} / (|Y_{\Delta_i}|)_{\text{SM}}$, $(|Y_{B-L}|)^{\text{f}}_{\text{SUSY}} / (|Y_{B-L}|)^{\text{f}}_{\text{SM}}$, $(|Y_{B-L}|)^{\text{non}-\text{f}}_{\text{SUSY}} / (|Y_{B-L}|)^{\text{non}-\text{f}}_{\text{SM}}$. Here, for example, $|Y_{B-L}|^{\text{f}}_{\text{SM}}$ is the lepton asymmetry in the case non-SUSY+flavor, and $|Y_{B-L}|^{\text{non}-\text{f}}_{\text{SUSY}}$ is the lepton asymmetry in the case SUSY+non-flavor, etc. K^{SUSY} can be written as $K^{\text{SUSY}} \sim 51/\eta_1$.

- (20)]:

$$K_e^{\text{SUSY}} = \frac{\Gamma^{\text{SUSY}}_{N_1 \to l_e H}(T=0)}{H(T=M_1)} \simeq 1.9 \left(\frac{5.7 \times 10^7 \text{ GeV}}{M_1} \right), \qquad (25)$$

$$K_\mu^{\text{SUSY}} = \frac{\Gamma^{\text{SUSY}}_{N_1 \to l_\mu H}(T=0)}{H(T=M_1)} \simeq 8.8 \left(\frac{5.7 \times 10^7 \text{ GeV}}{M_1} \right), \qquad (26)$$

$$K_\tau^{\text{SUSY}} = \frac{\Gamma^{\text{SUSY}}_{N_1 \to l_\tau H}(T=0)}{H(T=M_1)} \simeq 40 \left(\frac{5.7 \times 10^7 \text{ GeV}}{M_1} \right). \qquad (27)$$

As in the non-flavor case, these K factors become $\sqrt{2}$ times larger than in the non-SUSY case. These corrections make the washout of each asymmetry stronger compared with non-SUSY case and the SUSY effects become weak especially in the strong washout regime. However, even if $K^{\text{SM}} > 1$, some of the K_i^{SUSY} can be smaller than one, and therefore, the washout effect for

the flavor i is negligible. Then the supersymmetric contribution become sizable. Consequently, the lepton asymmetry generation is sufficiently boosted compared with the case of the non-flavor especially when $K^{\mathrm{SM}} > 1$.

Figure 6 shows the evolutions of total lepton asymmetry $|Y_{B-L}|$ and partial asymmetries of each lepton flavor $|Y_{\Delta_i}|$ for $M_1/M_1^0 = 16$. To understand the importance of the flavor effects in SUSY calculation, we make Figure 7 in which the ratios of SUSY lepton asymmetry to non-SUSY lepton asymmetry are plotted. First of all, the figure shows that SUSY enhancement factor is larger in the weak washout regime than in the strong washout regime as explained in the previous subsection. Next, the figure shows that the SUSY enhancement factor with the flavor effect is larger than without the flavor effect. Especially, it is important that even in the strong washout regime, the SUSY enhancement factor $|Y_{B-L}|^{\mathrm{f}}_{\mathrm{SUSY}}/|Y_{B-L}|^{\mathrm{f}}_{\mathrm{SM}}$ become sizable due to the enhancements of the muon and electron asymmetry, because $|Y_{B-L}|^{\mathrm{f}}_{\mathrm{SM}}$ has already been fairly larger than $|Y_{B-L}|^{\mathrm{non-f}}_{\mathrm{SM}}$ in the strong washout regime.

We could confirm the successful baryon asymmetry in the SUSY+flavor case, namely, in a realistic situation of the $E_6 \times U(1)_A$ GUT model. It is sufficient to take the lightest RH neutrino mass $M_1 \sim 16 \times M_1^0 \sim 9 \times 10^8$ GeV for the observed baryon asymmetry. It is important for this calculation that all components of neutrino Yukawa matrix are determined by the symmety in the $E_6 \times U(1)_A$ GUT and we can integrate the flavor effects on the lepton asymmetry.

5   Summary and Discussion

We have investigated the thermal leptogenesis in the $E_6 \times U(1)_A$ GUT model in which realistic quark and lepton masses and mixings are obtained and the doublet-triplet splitting problem is solved with natural assumption that all interactions including higher dimensional interactions are introduced with $O(1)$ coefficients. Each of three fundamental representations **27** includes two SM singlet fields, $S(\mathbf{1'})$ and $N_R^c(\mathbf{1})$, and these singlet fields play a role of RH neutrinos N_α ($\alpha = 1, 2, ..., 6$). One of the aim of this work is to show a sufficient lepton asymmetry is generated by the CP asymmetric decays of the lightest RH neutrino. In the model, Majorana masses of the RH neutrinos M_α and the neutrino Yukawa couplings $Y_{\alpha i}$ are determined by the $U(1)_A$ symmetry. So we can calculate the lepton asymmetry, but unfortunately the naive calculation results in too small abundance of the lepton asymmetry. Actually, the lightest RH neutrino mass is around 6×10^7 GeV, which is smaller than the Ibarra's lower bound 10^{8-9} GeV. Moreover, the factor K and the CP asymmetry $\epsilon_{N_1}^{SM}$ are evaluated as $K \sim 40$ and $\epsilon_{N_1}^{\mathrm{SM}} \sim 5 \times 10^{-9}$.

Therefore, the lepton asymmetry is washed out strongly in this scenario, and even with $K \sim 1$, the $\epsilon_{N_1}^{\mathrm{SM}}$ is too small to obtain the sufficient number of lepton asymmetry.

We have shown that a key ingredient for successful leptogenesis is the enhancement of RH neutrino masses. The model can include a large number of higher dimensional interactions which, and these interaction terms yield additional Majorana masses after developing the VEVs of negatively $U(1)_A$ charged fields. The enhancements of the RH neutrino masses enhance the CP asymmetry $\epsilon \propto M_1$ and make the decay parameter $K \propto 1/M_1$ smaller to be most efficient value $K \sim 1$. How large enhancement factor is required for the sufficient leptogenesis? To answer this question, we have calculated the lepton asymmetry including the effects of SUSY and flavor in the final state of the CP asymmetric decay. The result is that the enhancement factor $16 - 17$ is sufficient for the successful leptogenesis. About 300 mass terms are sufficient to obtain this enhancement factor, and this number looks not to be difficult to be obtained even in the E_6 GUT model. It is important that such enhancement of the lightest RH neutrino mass does not change the neutrino physics at the low energy scale. This is because the $E_6 \times U(1)_A$ GUT has six RH neutrinos which induces the same order of the amplitude of all elements of the light neutrino mass matrix.

We have calculated the lepton asymmetry in the $E_6 \times U(1)_A$ model in following four cases: (i) non-SUSY+non-flavor (ii) non-SUSY+flavor (iii) SUSY+non-flavor (iv) SUSY+flavor. These calculation has shown that both the effects of lepton flavor and SUSY are important. It is known that in the strong washout regime lepton flavor effect becomes sizable, though SUSY contribution is not so large. We have shown that SUSY contribution becomes important even in the strong washout regime if lepton flavor effect is included. The essential point is that even in the strong regime $K^{\mathrm{SM}} > 1$, the washout effects of the muon and/or the electron can become weak, and therefore these lepton number abundances become sizable.

Of course, the obtained result for the enhancement factor $16 - 17$ ($M_1 \sim 9 \times 10^8$ GeV) for the sufficient leptogenesis is dependent on the various parameters and even on the $O(1)$ parameters. For example, we have fixed the coefficient of $\epsilon_{N_1}^{\mathrm{SM}}$ in Eq. (13) as two in our calculation. Since the final lepton asymmetry is proportional to this $\epsilon_{N_1}^{\mathrm{SM}}$ parameter, the dependence can be read from the Fig. 1. When the coefficient is one, the required enhancement factor becomes around 25 ($M_1 \sim 1.4 \times 10^9$ GeV), and therefore, $O(600)$ mass terms are needed. When the coefficient is four, it becomes around 10 ($M_1 \sim 6 \times 10^8$ GeV), which is required $O(100)$ mass terms. Therefore, we will not predict the mass of the lightest RH neutrino for sufficient leptogenesis, because it depends on the various parameters. An important thing is that

the $E_6 \times U(1)_A$ GUT can explain the baryon asymmetry in the universe.

It is not plausible to produce the sufficient lepton number in the $SO(10) \times U(1)_A$ GUT[7] by the enhancement of the lightest RH neutrino. Since the number of the RH neutrinos is three in $SO(10)$ model, the other two neutrinos must have the masses expected by the symmetry. However, the difference between the $U(1)_A$ charges of the lightest RH neutrino and the second lightest RH neutrino mass terms is just two, and therefore, it is not reasonable to expect that the lightest RH neutrino has $O(10)$ times larger mass than the second lightest RH neutrino. However, since the $E_6 \times U(1)_A$ has six RH neutrinos and the difference between the $U(1)_A$ charges of the lightest RH neutrino and the forth lightest RH neutrino mass terms is six, it is plausible that the lightest RH neutrino has $O(100)$ times larger number of mass terms than the forth lightest RH neutrino. The observed baryon asymmetry in our universe may be an indirect signature of E_6 GUT.

Acknowledgments

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